

VI. *An Account of a Book intituled, A Treatise of Fluxions, in Two Books, by Colin M^c Laurin, A. M. Professor of Mathematics in the University of Edinburgh, and Fellow of the ROYAL SOCIETY, 4^{to}. in Two Volumes, Pages 763.*

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THE Author's first Design, in composing this Treatise, was to establish the Method of Fluxions on Principles equally evident and unexceptionable with those of the antient Geometricians, by Demonstrations deduced after their Manner, in the most rigid Form, and by illustrating the more abstruse Parts of the Doctrine, to vindicate it from the Imputation of Uncertainty or Obscurity. But he has likewise comprehended in this Work the Application of Fluxions to the most important geometrical and philosophical Inquiries. It consists of an Introduction, and Two Books. In the Introduction he gives an Abstract of the Discoveries of the Antients in the higher Parts of Geometry, with Observations on their Method, and those that first succeeded to it. The First Book treats of Fluxions in a geometrical Method, and the Second treats of the Computations.

In the Introduction we have an Abstract not only of the Discoveries of the Antients in the higher Parts of Geometry, but likewise of their Demonstrations. After an Account of the Propositions of this kind, that are to be found in the Twelfth Book of *Euclid*,

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there follows a Summary of what is most material in the Treatises of *Archimedes*, concerning the Sphere and Cylinder, Conoids and Spheroids, the Quadrature of the *Parabola* and the spiral Lines. The Demonstrations are not precisely in the same Form as those of *Archimedes*, but are often illustrated from the elementary Propositions concerning the Cone, or Corollaries from them, after the Example of *Pappus*, (*Coll. Math.* Prop. 21st, Lib. 4.) from whom a Proposition is demonstrated, and rendered more general, concerning the Area of the Spiral that is generated on a spherical Surface by the Composition of Two uniform Motions analogous to those by which the Spiral of *Archimedes* is described on a Plane. This Area, though a Portion of a curve Surface, is found to admit of a perfect Quadrature, and this Proposition concludes the Abstract. He takes occasion from these Theorems to demonstrate some Properties of the Conic Sections, that are not mentioned by the Writers on that Subject; and there are more of this kind described in the XIth and XIVth Chapters of the First Book.

It is known, that if a Parallelogram, circumscribed about a given Ellipse, have its Sides parallel to the conjugate Diameters, then shall its Area be of an invariable or given Magnitude, and equal to the Rectangle contained by the Axes of the Figure; but this is only a Case of a more general Proposition. For if, upon any Diameter produced without the Ellipse, you take Two Points, one on each Side of the Centre at equal Distances from it, and the Four Tangents be drawn from these Points to the Ellipse, those Tangents shall form a Parallelogram, which is
always

always of a given or invariable Magnitude, when the Ellipse is given, if the *Ratio* of those Distances to the Diameter be given; and when the *Ratio* of those Distances to the Semidiameter is that of the Diagonal of a Square to the Side, (or of $\sqrt{2}$ to 1) the Parallelogram has its Sides parallel to conjugate Diameters. It is likewise shown here, how the Triangles, *Trapezia*, or Polygons of any kind are determined, which, circumscribed about a given Ellipse, are always of a given Magnitude.

There is also a general Theorem concerning the *Frustum* of a Sphere, Cone, Spheroid, or Conoid, terminated by parallel Planes, when compared with a Cylinder of the same Altitude on a Base equal to the middle Section of the *Frustum* made by a parallel Plane. The Difference betwixt the *Frustum* and the Cylinder is always the same in different Parts of the same, or of similar Solids, when the Inclination of the Planes to the Axis, and the Altitude of the *Frustum*, are given. This Difference vanishes in the parabolic Conoid. It is the same in all Spheres; being equal to half the Content of a Sphere of a Diameter equal to the Altitude of the *Frustum*. In the Cone it is One-fourth of the Content of a similar Cone of the same Height with the *Frustum*; and in other Figures it is reduced to the Difference in the Cone.

In the Remarks on the Method of the Antients, the Author observes, that they established the higher Parts of their Geometry on the same Principles as the Elements of the Science, by Demonstrations of the same kind; that they seem to have been careful not to suppose any thing to be done, till by a previous Problem

blem they had shown how it was to be performed: Far less did they suppose any thing to be done, that cannot be conceived to be possible, as a Line or Series to be actually continued to Infinity, or a Magnitude to be diminished till it become infinitely less than it was. The Elements into which they resolved Magnitudes were always finite, and such as might be conceived to be real. Unbounded Liberties have been introduced of late, by which Geometry (wherein every thing ought to be clear) is filled with Mysteries, and Philosophy is likewise perplexed. Several Instances of this kind are mentioned. The Series 1, 2, 3, 4, 5, 6, 7, &c. is supposed by some to be actually continued to Infinity; and, after such a Supposition, we are puzzled with the Question, Whether the Number of finite Terms in such a Series is finite or infinite. In order to avoid such Suppositions, and their Consequences, the Author chose to follow the Antients in their Method of Demonstration as much as possible. Geometry has been always considered as our surest Bulwark against the Subtleties of the Sceptics, who are ready to make use of any Advantages that may be given them against it*; and it is important, not only that the Conclusions in Geometry be true, but likewise that their Evidence be unexceptionable. However, he is far from affirming, that the Method of Infinitesimals is without Foundation, and afterwards endeavours to justify a proper Application of it.

The Grounds of the Method of Fluxions are described in Chap. I. Book I. and again in Chap. I.

* See Bayle's Dictionary, Article *Zeno*.

Book II. In the former, Magnitudes are conceived to be generated by Motion, and the Velocity of the generating Motion is the Fluxion of the Magnitude. Lines are supposed to be generated by the Motion of Points. The Velocity of the Point that describes the Line is its Fluxion, and measures the Rate of its Increase or Decrease. Other Magnitudes may be represented by Lines that increase or decrease in the same Proportion with them; and their Fluxions will be in the same Proportion as the Fluxions of those Lines, or the Velocities of the Points that describe them. When the Motion of a Point is uniform, its Velocity is constant, and is measured by the Space which is described by it in a given Time. When the Motion varies, the Velocity at any Term of the Time is measured by the Space which would be described in a given Time, if the Motion was to be continued uniformly from that Term without any Variation. In order to determine that Space, and consequently the Velocity which is measured by it, Four Axioms are proposed concerning variable Motions, Two concerning Motions that are accelerated, and Two concerning such as are retarded. The First is, That the Space described by an accelerated Motion is greater than the Space which would have been described in the same Time, if it had not been accelerated, but had continued uniform from the Beginning of the Time. The Second is, That the Space which is described by an accelerated Motion, is less than the Space which is described in an equal Time by the Motion which is acquired by that Acceleration continued afterwards uniformly. By these, and Two similar Axioms concerning retarded Mo-

tions,

tions, the Theory of Motion is rendered applicable to this Doctrin with the greatest Evidence, without supposing Quantities infinitely little, or having recourse to prime or ultimate *Ratios*. The Author first demonstrates from them all the general Theorems concerning Motion, that are of Use in this Doctrin; as that when the Spaces described by Two variable Motions are always equal, or in a given *Ratio*, the Velocities are always equal, or in the same given *Ratio*; and conversely, when the Velocities of Two Motions are always equal to each other, or in a given *Ratio*, the Spaces described by those Motions in the same Time are always equal, or in that given *Ratio*; that when a Space is always equal to the Sum or Difference of the Spaces described by Two other Motions, the Velocity of the First Motion is always equal to the Sum or Difference of the Velocities of the other Motions; and conversely, that when a Velocity is always equal to the Sum or Difference of Two other Velocities, the Space described by the First Motion is always equal to the Sum or Difference of the Spaces described by these Two other Motions. In comparing Motions in this Doctrin, it is convenient and usual to suppose one of them uniform; and it is here demonstrated, that if the Relation of the Quantities be always determined by the same Rule or Equation, the *Ratio* of the Motions is determined in the same manner, when both are supposed variable. These Propositions are demonstrated strictly by the same Method which is carried on in the ensuing Chapters for determining the Fluxions of the Figures.

In Chap. II. a Triangle that has Two of its Sides given in Position, is supposed to be generated by an Ordinate moving parallel to itself along the Base. When the Base increases uniformly, the Triangle increases with an accelerated Motion, because its successive Increments are *Trapezia*, that continually increase. Therefore, if the Motion with which the Triangle flows, was continued uniformly from any Term for a given Time, a less Space would be described by it than the Increment of the Triangle which is actually generated in that Time by Axiom I. but a greater Space than the Increment which was actually generated in an equal Time preceding that Term, by Axiom II. and hence it is demonstrated, that the Fluxion of the Triangle is accurately measured by the Rectangle contained by the corresponding Ordinate of the Triangle, and the right Line which measures the Fluxion of the Base. The Increment which the Triangle acquires in any Time, is resolved into Two Parts; that which is generated in consequence of the Motion with which the Triangle flows at the Beginning of the Time, and that which is generated in consequence of the Acceleration of this Motion for the same Time. The latter is justly neglected in measuring that Motion (or the Fluxion of the Triangle at that Term), but may serve for measuring its Acceleration, or the Second Fluxion of the Triangle. The Motion with which the Triangle flows, is similar to that of a Body descending in free Spaces by an uniform Gravity, the Velocity of which, at any Term of the Time, is not to be measured by the Space described by the Body in a given Time, either before or after that Term, because the

Motion continually increases, but by a Mean between these Spaces.

When the Sides of a Rectangle increase or decrease with uniform Motions, it may be always considered as the Sum or Difference of a Triangle and *Trapezium*; and its Fluxion is derived from the last Proposition. If the Sides increase with uniform Motions, the Rectangle increases with an accelerated Motion; and in measuring this Motion at any Term of the Time, a Part of the Increment of the Rectangle, that is here determined, is rejected, as generated in consequence of the Acceleration of that Motion.

The Fluxions of a curvilinear Area (whether it be generated by an Ordinate moving parallel to itself, or by a Ray revolving about a given Centre) and of the Solid, generated by the Area revolving about the Base, are determined by Demonstrations of the same kind; and when the Ordinates of the Figure increase, the Increment of the Area is resolved in like manner into Two Parts, one of which is only to be retained in measuring the Fluxion of the Area, the other being rejected as generated in consequence of the Acceleration of the Motion with which the Figure flows. An Illustration of Second and Third Fluxions is given by resolving the Increment of a Pyramid or Cone into the several respective Parts that are conceived to be generated in consequence of the First, Second, and Third Fluxions of the Solid, when the Axis is supposed to flow uniformly.

In Chap. V. a Series of Lines in Geometrical Progression are represented by an easy Construction. The First Term being supposed invariable, and the Second to increase uniformly, all the subsequent
Terms

Terms increase with accelerated Motions. The Velocities of the Points that describe those Lines being compared, it is demonstrated from the Axioms by common Geometry, that the Fluxions of any Two Terms are in a *Ratio* compounded of the *Ratio* of the Terms, and of the *Ratio* of the Numbers that express how many Terms precede them in the Progression.

In the VIth Chapter, the Nature and Properties of Logarithms are described after the celebrated Inventor; and it is observed, that he made use of the very Terms *Fluxus* and *Fluat* on this Occasion. A Line is said to increase or decrease *proportionally*, when the Velocity of the Point, that describes it, is always as its Distance from a certain Term of the Line; and if in the mean time another Point describes a Line with a certain uniform Motion, the Space described by the latter Point is always the Logarithm of the Distance of the former from the given Term. Hence the Fluxion of this Distance is to the Fluxion of its Logarithm as that Distance is to an invariable Line; and the Fluxions of the Quantities that have their Logarithms in an invariable *Ratio*, are to each other in a *Ratio* compounded of this invariable *Ratio*, and of the *Ratio* of the Quantities themselves. Some Propositions are demonstrated, that relate to the Computation of Logarithms; but this Subject is prosecuted farther in the Second Book. The Logarithmic Curve is here described, with the Analogy betwixt Logarithms and Hyperbolic *Ratios*.

In the VIIth Chapter, after a general Definition of Tangents, it is demonstrated, that the Fluxions of the Base, Ordinate, and Curve, are in the same Proportion

tion to each other, as the Sides of a Triangle respectively parallel to the Base, Ordinate, and Tangent. When the Base is supposed to flow uniformly, if the Curve be convex towards the Base, the Ordinate and Curve increase with accelerated Motions; but their Fluxions at any Term are the same as if the Point which describes the Curve had proceeded uniformly from that Term in the Tangent there. Any further Increment which the Ordinate or Curve acquires, is to be imputed to the Acceleration of the Motions with which they flow. A Ray that revolves about a given Centre, being supposed to meet any Curve and an Arc of a Circle described from the same Centre, the Fluxions of the Ray, Curve, and circular Arc, are compared together; and several other Propositions concerning Tangents are demonstrated from the Axioms. The next Chapter treats of the Fluxions of curve Surfaces in a similar manner.

The IXth Chapter treats chiefly of the greatest and least Ordinates of Figures, and of the Points of contrary Flexure and Cuspids. The Fluxion of the Base being given, when the Fluxion of the Ordinate vanishes, the Tangent becomes parallel to the Base, and the Ordinate most commonly is a *Maximum* or *Minimum*, according to the Rule given by Authors upon this Subject. But if the Second Fluxion of the Ordinate vanish at the same time, and the Third Fluxion be real, this Rule does not hold, for the Ordinate is in that Case neither a *Maximum* nor *Minimum*. If the First, Second, and Third Fluxions vanish, and the Fourth Fluxion be real, the Ordinate is a *Maximum* or *Minimum*. The general Rule demonstrated in this Chapter, and again in the last Chapter of the

Second

Second Book, is, that when the First Fluxion of the Ordinate, with its Fluxions of any subsequent successive Orders, vanish, and the Number of all these Fluxions that vanish is odd, then the Ordinate is a *Maximum* or *Minimum*, according as the Fluxion of the next Order to these is negative or positive. The Ordinate passes through a Point of contrary Flexure, when its Fluxion becomes a *Maximum* or *Minimum*, supposing the Curve to be continued on both Sides of the Ordinate. Hence the common Rule for finding the Points of contrary Flexure is corrected in a similar manner. Such a Point is not always formed when the Second Fluxion of the Ordinate vanishes; for if its Third Fluxion likewise vanishes, and its Fourth Fluxion be real, the Curve may have its Cavity turned all one Way. The same is to be said, when its Fluxions of the subsequent successive Orders vanish, if the Number of all those that vanish be even. Other Theorems are subjoined relating to this Subject.

The Xth Chapter treats of the Asymptotes of Lines, the Areas bounded by them and the Curves, the Solids generated by these Areas, of spiral Lines, and the Limits of the Sums of Progressions. The Analogy there is betwixt these Subjects, induced the Author to treat of them in one Chapter, and illustrate them by one another. He begins with Three of the most simple Instances of Figures that have Asymptotes. In the common Hyperbola, the Ordinate is reciprocally as the Base, and therefore decreases while the Base increases, but never vanishes, because the Rectangle contained by it and the Base is always a given Area, and it is assignable at any assignable Distance, how

how great soever. The Points of the Conchoid are determined by drawing right Lines from a given Centre, and upon these produced from the Asymptote, taking always a given right Line; so that the Curve never meets the Asymptote, but continually approaches to it, because of the greater and greater Obliquity of this right Line. The Third is the Logarithmic Curve, wherein the Ordinates, at equal Distances, decrease in Geometrical Proportion, but never vanish, because each Ordinate is in a given *Ratio* to the preceding Ordinate. Geometrical Magnitude is always understood to consist of Parts; and to have no Parts, or to have no Magnitude, are considered as equivalent in this Science *. There is, however, no Necessity for considering Magnitude as made up of an infinite Number of small Parts; it is sufficient, that no Quantity can be supposed to be so small, but it may be conceived to be diminished further; and it is obvious, that we are not to estimate the Number of Parts that may be conceived in a given Magnitude, by those which in particular determinate Circumstances may be actually perceived in it by Sense; since a greater Number of Parts become visible in it by varying the Circumstances in which it is perceived.

It is hardly possible to give a tolerable Extract of this or the following Chapters, without Diagrams and Computations: We shall therefore observe only, that after giving some plain and obvious Instances, wherein a Quantity is always increasing, and yet never

* See *Euclid's Elements*, Def. I. Lib. I.

amounts to a certain finite Magnitude (as, while the Tangent increases, the Arc increases, but never amounts to a Quadrant); this is applied successively to the several Subjects mentioned in the Title of the Chapter. Let the Figure be concave towards the Base, and suppose it to have an Asymptote parallel to the Base; in this Case the Ordinate always increases while the Base is produced, but never amounts to the Distance between the Asymptote and the Base. In like manner a curvilinear Area, in a Second Figure, may increase, while the Base is produced, and approach continually to a certain finite Space, but never amount to it: This is always the Case, when the Ordinate of this latter Figure is to a given right Line, as the Fluxion of the Ordinate of the former is to the Fluxion of the Base; and of this various Examples are given. A Solid may increase in the same manner, and yet never amount to a given Cube or Cylinder, when the Square of the Ordinate of the latter Figure is to a given Square, as the Fluxion of the Ordinate of the first Figure is to the Fluxion of the Base. A Spiral may in like manner approach to a Point continually, and yet in any Number of Revolutions never arrive at it; and there are Progressions of Fractions that may be continued at Pleasure, and yet the Sum of the Terms may be always less than a given Number. Various Rules are demonstrated, and illustrated by Examples, for determining when a Figure has an Asymptote parallel or oblique to the Base; when the Area terminated by the Curve and the Asymptote has a Limit which it never exceeds, or may be produced till it surpasses any assignable Space; when the Solid generated by that Area, the

the Surface generated by the Perimeter of the Curve, the spiral Area generated by the revolving Ray, the spiral Line itself, or the Sum of the Terms of a Progression, have such Limits or not; and for measuring those Limits. The Author insists on these Subjects, the rather that they are commonly described in very mysterious Terms, and have been the most fertile of Paradoxes of any Parts of the higher Geometry. These Paradoxes, however, amount to no more than this: That a Line or Number may be continually acquiring Increments, and those Increments may decrease in such a manner, that the whole Line or Number shall never amount to a given Line or Number. The Necessity of admitting this is obvious enough, and is here shewn from the Nature of the most common geometrical Figures in Art. 292, 293, &c. and from any Series of Fractions that decrease continually, in Art. 354, 355, &c.

The XIth Chapter treats of the Curvature of Lines, its Variation, the Degrees of Contact of the Curve and Circle of Curvature, and of various Problems that depend on the Curvature of Lines. This Subject is treated fully, because of its extensive Usefulness, and because in this consists one of the greatest Advantages of the modern Geometry above that of the Antients. The Author on this, as former Occasions, begins by premising the necessary Definitions. Curve Lines touch each other in a Point, when the same right Line is their common Tangent at that Point; and that which has the closest Contact with the Tangent, or passes betwixt it and the other Curve through the Angle of Contact formed by them, being less inflected from the Tangent, is therefore less curve. Thus a
greater

greater Circle has a less Curvature than a lesser Circle; and since the Curvature of Circles may be varied indefinitely, by enlarging or diminishing their Diameters, they afford a Scale by which the Curvature of other Lines may be measured. As the Tangent is the right Line which touches the Arc so closely, that no other right Line can be drawn between them; so the Circle of Curvature is that which touches the Curve so closely, that no other Circle can be drawn through the Point of Contact between them. As the Curve is separated from its Tangent in consequence of its Flexure or Curvature, so it is separated from the Circle of Curvature in consequence of the Variation of its Curvature; which is greater or less, according as its Flexure from that Circle is greater or less.

The Tangent of the Figure being considered as the Base, a new Figure is imagined, whose Ordinate is a Third Proportional to the Ordinate and Base of the First. This new Figure determines the Chord of the Circle of Curvature by its Intersection with the Ordinate at the Point of Contact, and by the Tangent of the Angle in which it cuts that Circle, measures the Variation of Curvature. The less this Angle is, the closer is the Contact of the Curve and Circle of Curvature, of which there may be indefinite Degrees. When the Figure proposed is a conic Section, the new Figure is likewise a conic Section; and it is a right Line when the First Figure is a *Parabola*, and the Ordinates are parallel to the Axis; or when the First Figure is an *Hyperbola*, and the Ordinates are parallel to either Asymptote. Hence the Curvature and its Variation in a conic Section are determined by several Constructions; and, amongst other The-

orems, it is shewn, that the Variation of Curvature at any Point of a conic Section is as the Tangent of the Angle contained by the Diameter which passes through that Point, and by the Perpendicular to the Curve.

When the Ordinate at the Point of Contact is an Asymptote to the new Figure, the Curvature is less than in any Circle; and this is the Case in which it is said to be infinitely little, or the Ray of Curvature is said to be infinitely great. Of this kind is the Curvature at the Points of contrary Flexure in the Lines of the Third Order. When the new Figure passes through the Point of Contact, the Curvature is greater than in any Circle, or the Ray of Curvature vanishes; and in this Case the Curvature is said to be infinitely great. Of this kind is the Curvature at the Cuspids of the Lines of the Third Order.

As Lines which pass through the same Point have the same Tangent when the First Fluxions of the Ordinate are equal, so they have the same Curvature when the Second Fluxions of the Ordinate are likewise equal; and half the Chord of the Circle of Curvature that is intercepted between the Points wherein it intersects the Ordinate, is a Third Proportional to the right Lines that measure the Second Fluxion of the Ordinate and First Fluxion of the Curve, the Base being supposed to flow uniformly. When a Ray revolving about a given Point, and terminated by the Curve, becomes perpendicular to it, the First Fluxion of the Ray vanishes; and if its Second Fluxion vanishes at the same time, that Point must be the Centre of Curvature. The same is to be said when the angular Motion of the Ray about that
Point

Point is equal to the angular Motion of the Tangent of the Curve; as the angular Motion of the *Radius* of a Circle about its Centre is always equal to the angular Motion of the Tangent of the Circle. Thus the various Properties of the Circle suggest various Theorems for determining the Centre of the Curvature.

Because Figures are often supposed to be described by the Intersections of right Lines revolving about given Poles, Three Theorems are given in Prop. 18. 26. and 35. for determining the Tangents, Asymptotes, and Curvature of such Lines, from the Description, which are illustrated by Examples. A new Property of Lines of the Third Order is subjoined to Prop. 35. The Evolution of Lines is considered in Prop. 36. The Tangents of the *Evoluta* are the Rays of Curvature of the Line which is described by its Evolution; and the Variation of Curvature in the latter is measured by the *Ratio* of the Ray of Curvature of the former to the Ray of Curvature of the latter.

Sir *Isaac Newton*, in a Treatise lately published, measures the Variation of the Curvature by the *Ratio* of the Fluxion of the Ray of Curvature to the Fluxion of the Curve; and is followed by the Author, to avoid the Perplexity which a Difference in Definitions occasions to Readers, though he hints (in Art. 386.) that this *Ratio* gives rather the Variation of the Ray of Curvature, and that it might have been proper to have measured the Variation of Curvature rather by the *Ratio* of the Fluxion of the Curvature itself to the Fluxion of the Curve; so that the Curvature being inversely as the Ray of Curvature, and

consequently its Fluxion as the Fluxion of the Ray itself directly, and the Square of the Ray inverfely, its Variation would have been directly as the Measure of it, according to Sir *Isaac Newton's* Definition, and inverfely as the Square of the Ray of Curvature: According to this Explication, it would have been measured by the Angle of Contact contained by the Curve and Circle of Curvature, in the same manner as the Curvature itself is measured by the Angle of Contact contained by the Curve and Tangent. The Ground of this Remark will better appear from an Example: According to Sir *Isaac Newton's* Explication, the Variation of Curvature is uniform in the Logarithmic Spiral, the Fluxion of the Ray of Curvature in this Figure being always in the same *Ratio* to the Fluxion of the Curve; and yet while the Spiral is produced, though its Curvature decreases, it never vanishes; which must appear strange to such as do not attend to the Import of his Definition.—It is easy, however, to derive one of these Measures of this Variation from the other, and because Sir *Isaac Newton's* is (generally speaking) assigned by more simple Expressions, the Author has the rather conformed to it in this Treatise, but thought it necessary to give the Caution we have mentioned.

The greatest Part of this Chapter is employed in treating of useful Problems, that have a Dependence on the Curvature of Lines. First, the Properties of the Cycloid are briefly demonstrated, with the Application of this Doctrine to the Motion of Pendulums, by shewing that when the Motion of the generating Circle along the Base is uniform, and therefore may measure the Time, the Motion of the Point that de-

describes the Cycloid, is such as would be acquired by a heavy Body descending along the cycloidal Arc, the Axis of the Figure being supposed perpendicular to the Horizon. In the next place, the Caustics, by Reflexion and Refraction, are determined. If Perpendiculars be always drawn from the radiating Point to the Tangents of the Curve, and a new Curve be supposed to be the *Locus* of the Intersections of the Perpendiculars and Tangents, then the Line, by the Evolution of which that new Curve can be described, is similar and similarly situated to the Caustic by Reflexion. The Doctrine of centripetal Forces is treated at length from Art. 416. to 493.

First, a Body is supposed to descend freely by its Gravity in a vertical Line; and because the Gravity is the Power which accelerates the Motion of the Body, it must be measured by the Fluxion of its Velocity, or the Second Fluxion of the Space described by it. When the vertical Line is supposed to move parallel to itself with an uniform Motion, the Body will descend in it in the same manner as before; and the Gravity will be still measured by the Second Fluxion of the Descent, or the Second Fluxion of the Ordinate of the Curve that is traced in this Case by the Body on an immoveable Plain, and therefore is as the Square of the Velocity (which is measured by the Fluxion of the Curve) directly, and the Chord of the Circle of Curvature that is in the Direction of the Gravity inversely, by a Proposition mentioned above. When the Gravity acts uniformly, and in parallel Lines, the Projectile, in describing any Arc, falls below the Tangent drawn at the Beginning of the Arc, as much as if it had fallen perpendicu-
larly

larly in the Vertical; and the Time being given, the Gravity may be measured by the Space which is the Subtense of the Angle of Contact. In other Cases, when the Gravity varies, or its Direction changes, it may be measured at any Point by the Subtense of the Angle of Contact, that would have been generated in a given Time, if the Gravity had continued to act uniformly in parallel Lines from that Term, that is, by the Subtense of the Angle of Contact in the Parabola that has its Diameter in the Direction of the Force, and has the closest Contact with the Curve; which leads us to the same Theorem as before.

In general, let the Gravity (that results from the Composition of any Number of centripetal Forces, which are supposed to act on the Body in one Plane) be resolved into a Force parallel to the Base; then the former shall be measured by the Second Fluxion of the Ordinate, and the latter by the Second Fluxion of the Base, the Time being supposed to flow uniformly, so that the Velocity of the Body may be measured by the Fluxion of the Curve. When the Trajectory is not in one Plane, the Force is resolved in a similar manner into Three Forces, which are measured by Three Second Fluxions analogous to them.

Whether the Body move in a Void, or in a Medium that resists its Motion; the Gravity that results from the Composition of the centripetal Forces which act upon the Body, is always as the Square of its Velocity directly, and the Chord of the Circle of Curvature that is in the Direction of the Gravity inversely.

When

When a Body describes any Trajectory in a Void or in a Medium, by a Force directed to One given Centre, the Velocity at any Point of the Trajectory is to the Velocity by which a Circle could be described in a Void about the same Centre, at the same Distance, by the same Gravity, in the subduplicate *Ratio* of the angular Motion of the Ray drawn always from the Body to the Centre, to the angular Motion of the Tangent of the Trajectory: And, if there be no Resistance, the Velocity in the Trajectory at any Point, is the same that would be acquired by the Body, if it was to fall from that Point through One-fourth of the Chord of the Circle of Curvature that is in the Direction of the Gravity, and the Gravity at that Point was to be continued uniformly during its Descent.

If the centripetal Force be inversely as any Power of the Distance whose Exponent is any Number m greater than Unit, there is a certain Velocity (*viz.* that which is to the Velocity in a Circle at the same Distance as $\sqrt{2}$ to $\sqrt{m-1}$) which would be just sufficient to carry off the Body upwards in a vertical Line, so as that it should continue to ascend for ever, and never return towards the Centre. If the Body be projected in any other Direction with the same Velocity, it will describe a Trajectory which is here constructed: It is a *Parabola* when $m=2$, a Logarithmic Spiral when $m=3$, an Epicycloid when $m=4$, a Circle that passes through the Centre of the Forces when $m=5$, and the *Lemniscata* when $m=7$. In general, it is constructed by drawing a Perpendicular from the Centre of the Forces to a right Line given in Position, and any other Ray to the same right Line, then

then increasing or diminishing the Angle contained by this Ray and the Perpendicular in the given *Ratio* of 2 to the Difference between 3 and m , and increasing or diminishing the Logarithm of the Ray in the same given *Ratio*. The Trajectories described in analogous Cases by centrifugal Forces, are constructed in a similar manner. These are the Figures in which the Perpendicular, from a given Centre on the Tangent, is always as some Power of the Ray drawn from the same Centre to the Point of Contact, which are afterwards found to arise in the Resolution of the most simple Cases of Problems of various kinds.

When the Area described about the Centre of an Ellipse is given, the Subtense of the Angle of Contact, drawn through one Extremity of the Arc parallel to the Semidiameter drawn to the other Extremity, is in a given *Ratio* to this Semidiameter; and therefore, when an Ellipse is described by a Force directed towards the Centre, that Force is always as the Distance from the Centre. When the Force is directed toward the *Focus*, it is inversely as the Square of the Distance. And these Two Cases are considered particularly, because of their Usefulness in the true Theory of Gravity. To illustrate which, the Laws of centripetal Forces that would cause a Body to descend continually toward the Centre, or ascend from it, are distinguished from those which cause the Body to approach towards the Centre, and recede from it by turns. A Body approaches from the higher Apfid toward the Centre, when its Velocity is less than what is requisite to carry it in a Circle; and if its Velocity increase, while it descends, in a
higher

higher Proportion than the Velocities requisite to carry Bodies in Circles about the same Centre, the Velocity in the lower Part of the Curve may exceed the Velocity in a Circle at the same Distance, and thereby become sufficient to carry off the Body again. But while the Distance decreases, if the Velocities in Circles increase in the same or in a higher Proportion, than the Velocity in a Trajectory can increase, the Body must either continually approach toward the Centre, if it once begin to approach to it, or recede continually from the Centre, if it once begin to ascend from it; and this is the Case, when the centripetal Force increases as the Cube of the Distance decreases, or in a higher Proportion. But though, in such Cases, the Body approach continually towards the Centre, we are not to conclude, that it will always approach to it till it fall into it, or come within any given Distance; for it is demonstrated afterwards in Art. 879 and 880. that it may approach to the Centre for ever, in a Spiral that never descends to a given Circle described in the same Plane, and that it may recede from it for ever in a Spiral that never arises to a given Altitude. An Example of each Case is given when the centripetal Force is inversely as the Fifth Power of the Distance.

When the Trajectory is described in a *Medium*, let z be to a given Magnitude as the centripetal Force is to the Force by which the same Trajectory could be described in a Void; and if the Area be supposed to flow uniformly, the Resistance will be in the compound *Ratio* of the Fluxion of z , and of the Fluxion of the Curve; and the Density of the Medium (supposing the Resistance to be in the compound *Ratio*

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of the Density and of the Square of the Velocity) shall be as the Fluxion of the Logarithm of z directly, and the Fluxion of the Curve inversely. Hence, when any Figure that can be described in a Void by a Force that varies according to any Power of the Distance from the Centre, is described in a Medium, the Density of the Medium must be inversely as the Tangent of the Figure bounded by a Perpendicular at the Centre to the Ray drawn from it to the Point of Contact.

After giving some Properties of the Trajectories, that are described by a Body when it gravitates in right Lines perpendicular to a given Surface, and their Application to optical Uses, the Author proceeds to consider the Motion of a Body that gravitates towards several Centres. In such Cases, that Surface is said to be horizontal, which is always perpendicular to the Direction of the Gravity that results from the Composition of the several Forces; and it is shewn, that the Velocity which is acquired by descending from one horizontal Surface to another, is always the same (whether the Body move in right Lines, or in any Curves); the Square of which is measured by the Aggregate of several Areas which have the Distances from the respective Centres for their Bases, and right Lines proportional to the Forces at these Distances for their Ordinates.

The Force which acts upon the Moon is resolved into a Force perpendicular to the Plane of the Ecliptic, and a Force parallel to it. This last is again resolved into that which is parallel to the Line of the *Syzigies*, and that which is parallel to the Line joining the Quadratures. The First measures the Se-
cond

cond Fluxion of the Distance of the Moon from that Plane, the Second and Third measure the Second Fluxions of her Distances from the Line of the Quadratures, and from the Line of the *Syzigies*, respectively. Hence a Construction is derived of the Trajectory which would be described by the Moon about the Earth, in consequence of their unequal Gravitation towards the Sun, if the Gravity of the Moon towards the Earth was as her Distance from it. From this a Computation is deduced of the Motion of the Nodes of the Moon, and of the Variation of the Inclination of the Plane of her Orbit, which we cannot describe here. It is sufficient to observe, that these Motions are found to agree nearly with those which have been deduced from other Theories, and from Astronomical Observations.

A Fluid being supposed to gravitate towards two given Centres with equal and invariable Forces, it is shewn, that the Figure of the Fluid must be that of an oblong Spheroid, and that those two Centres must be the *Foci* of the generating Ellipse. The Nature of the Figure is also shewn, when the Fluid gravitates towards several Centres, or when it revolves on its Axis; but these are mentioned briefly, because such Theories are of little or no Use for discovering the Figures of the Planets.

In the 12th Chapter, the Author proceeds to consider the more concise Methods, by which the Fluxions of Quantities are usually determined, and to deduce general Theorems more immediately applicable to the Resolution of Geometrical and Philosophical Problems. In the Method of Infinitesimals, the Element, by which any Quantity increases or decreases,

is supposed to become infinitely small, and is generally expressed by Two or more Terms, some of which become infinitely less than the rest, and therefore being neglected as of no Importance, the remaining Terms form, what is called the *Difference* of the Quantity proposed. The Terms that are neglected in this manner are the very same which arise in consequence of the Acceleration or Retardation of the generating Motion, during the infinitely small Time in which the Element is generated; and therefore these Differences are in the same *Ratio* to each other as the generating Motions or Fluxions. Hence the Conclusions in this Method are accurately true, without even an infinitely small Error, and agree with those that are deduced by the Method of Fluxions.

It is usual in this Method to consider a Curve as a Polygon of an infinite Number of Sides, which, being produced, give the Tangents of the Curve, and, by their Inclination to each other, measure its Curvature. But it is necessary in some Cases, if we would avoid Error, to resolve the Element of the Curve into several infinitely small Parts, or even sometimes into Infinitesimals of the Second Order; and Errors that might otherwise arise in its Application, may, with due Care, be corrected by a proper Use of this Method itself, of which some Instances are given. If we were to suppose, for Example, the least Arc that can be described by a Pendulum to coincide with its Chord, the Time of the Vibration derived from this Supposition will be found erroneous; but by resolving that Arc into more and more infinitely small Parts, we approach to the true
Time

Time in which it is described. By supposing the Tangent of the Curve to be the Production of the rectilineal Element of the Curve, the Subtense of the Angle of Contact is found equal to the Second Difference or Fluxion of the Ordinate; but in this Inquiry, the Tangent ought to be supposed to be equally inclined to the two Elements of the Curve that terminate at the Point of Contact; and then the Subtense of the Angle of Contact will be found equal to half the Second Difference of the Ordinate, which is its true Value.

Sir *Isaac Newton*, however, investigates the Fluxions of Quantities in a more unexceptionable manner. He first determines the finite simultaneous Increments of the Fluents, and, by comparing them, investigates the *Ratio* that is the Limit of the various Proportions which they bear to each other, while he supposes them to decrease together till they vanish. When the generating Motions are variable, the *Ratio* of the simultaneous Increments that are generated from any Term, is expressed by several Quantities, some of which arise from the *Ratio* of the generating Motions at that Term, and others from the subsequent Acceleration or Retardation of these Motions. While the Increments are supposed to be diminished, the former remain invariable, but the latter decrease continually, and vanish with the Increments; and hence the Limit of the variable *Ratio* of the Increments (or their ultimate *Ratio*) gives the precise *Ratio* of the generating Motions or Fluxions. Most of the Propositions in the preceding Chapters may be more briefly demonstrated by this Method, (of which several
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several Examples are given) and the Author makes always use of it in the Sequel of this Book.

It is one of the great Advantages of this Method, that it suggests general Theorems for the Resolution of Problems, which may be readily applied as there is occasion for them. Our Author proceeds to treat of these, and first of such as relate to the Centre of Gravity and its Motion. In any System of Bodies, the Sum of their Motions, estimated in a given Direction, is the same as if all the Bodies were united in their common Centre of Gravity. If the Motion of all the Bodies is uniform and rectilinear, the Centre of Gravity is either quiescent, or its Motion is uniform and rectilinear. When Action is equal to Reaction, the State of the Centre of Gravity is never affected by the Collisions of the Bodies, or by their attracting or repelling each other mutually. It is not, however, the Sum of the absolute Motions of the Bodies that is preserved invariable in consequence of the Equality of the Action and Reaction, as they seem to imagine, who tell us, that this Sum is unalterable by the Collisions of Bodies, and that this follows so evidently from the Equality of Action and Reaction, that to endeavour to demonstrate it would serve only to render it more obscure. On this Occasion the Author illustrates an Argument which he had proposed in a Piece that obtained the Prize proposed by the *Royal Academy of Sciences at Paris* in 1724. against the Mensuration of the Forces of Bodies by the Square of the Velocities, shewing that if this Doctrine was admitted, the same Power or Agent, exerting the same Effort, would produce more Force in the same Body when in a Space carried uniformly forwards,

forwards, than if the Space was at Rest ; or that Springs acting equally on Two equal Bodies in such a Space, would produce unequal Changes in the Forces of those Bodies.

Various Problems concerning the Collision of Bodies are resolved in a more general manner than usual. Mr. *Bernouilli* had determined the Motions when the Elasticity is perfect, and One Body strikes Two equal Bodies in Directions that form equal Angles with its Direction; or when there are any Number of Bodies impelled by it on one Side in various Directions, providing equal Bodies be impelled by it on the other Side, in Directions equally inclined to its own Direction. But the Problem is resolved here without these Limitations; some others of this kind are subjoined, and this Doctrine is applied for determining the Motions of Bodies that act upon each other while they descend by their Gravity.

The general Principle derived from these Inquiries is, that if there be no Collision, or sudden Communication of Motion from one Body to another, while they descend together, and in any case, if the Elasticity be perfect, the Sum of the Products, when each Body is multiplied by the Square of the Velocity acquired by it, is the same as if all the Bodies had descended freely from the same respective Altitudes to their several Places; only in collecting that Sum, if any Body is made to ascend, the Product of it multiplied by the Square of its Velocity is to be subducted: And if the Bodies be supposed to ascend from their Places with the respective Velocities acquired by them, then their common Centre of Gravity will rise to the same

same Level from which it descended. In other Cases, however, the Ascent of the Centre of Gravity will be less than its Descent, but is never greater.

After demonstrating the usual Rule for finding the Centre of Oscillation, the Author treats of the Motion of Water issuing from a cylindric Vessel. The Effect of the Gravitation of the whole Mass of Water is considered as Threefold. It accelerates, for some time at least, the Motion with which the Water in the Vessel descends; it generates the Excess of the Motion with which the Water issues at the Orifice above the Motion which it had in common with the rest of the Water; and it acts on the Bottom of the Vessel at the same time. Then supposing the last Two Parts of the Force to be in any invariable *Ratio* to each other, when the Diameters of the Base and Orifice are given, he determines by Logarithms the Velocity with which the Water issues at the Orifice; and shews that this Velocity will approach very near to its utmost Limit in an exceeding small Time. When the Water is supposed to be supplied in a Cylinder, so as to stand always at the same Altitude above the Orifice, there is an Analogy between the Acceleration of the Motion of the Water that issues at the Orifice, and the Acceleration of a Body that descends by its Gravity in a Medium which resists in the duplicate *Ratio* of the Velocity. For when the utmost Velocities, or Limits, are equal in those two Cases, the Time in which the issuing Water acquires any lesser Velocity, is to the Time in which the descending Body acquires the same Velocity as the Area of the Orifice to the Area of the Base; and if a cylindric Column be supposed to
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be erected on the Orifice equal to the Quantity of Water that issues at the Orifice in the former of those Times, the Height of this Column will be to the Space described by the descending Body in the latter Time, in the same *Ratio* as the Orifice to the Area of the Base. The *Ratio* of the Force that acts on the Bottom of the Vessel to the Force that generates the Motion of the Water issuing at the Orifice, is deduced from Sir *Isaac Newton's* Cataract, and is the same that follows from the Principle concerning the Equality of the Ascent and Descent of the Centre of Gravity, which was first applied to this Inquiry by Mr. *Daniel Bernouilli* Comment. *Acad. Petrop. Tom.* 2. But there are several Precautions to be taken in applying this Doctrine.

After some other Theorems concerning the Centre of Gravity, and several Observations concerning the Curvature of Lines, and the Angles of Contact; the Author represents four general Propositions in one View, that the Analogy between them may appear. The First gives the Property of the Trajectories that are described by any centripetal Forces, how variable soever these Forces, or their Directions, may be. The Second gives a like general Property of the Lines of swiftest Descent. The Third gives the Property of the Line that is described in less time than any other of an equal Perimeter. And the Fourth gives the Property of the Figure that is assumed by a flexible Line or Chain, in consequence of any such Forces acting upon it. If we suppose a Body to set out from any Point in the Trajectory, or in the Line of swiftest Descent, with the Velocity which it has acquired there, and to move in the right Line which is the

Direction of the Gravity, that results from the Composition of the centripetal Forces, then shall its Velocity, and its Distance from the Point where the Perpendicular from the Centre of Curvature meets that right Line, flow *proportionally*, *i. e.* the Fluxion of the Velocity (or of the right Line that measures it) shall be to the Velocity as the Fluxion of that Distance is to the Distance. When the Velocity and Direction of the Motion is the same in the Line of swiftest Descent as in the Trajectory, their Curvature is the same. Thus in the common Hypothesis of Gravity, the Curvature in the Cycloid, the Line of swiftest Descent, is the same as the *Parabola* described by a Projectile, if the Velocities in those Lines be equal, and their Tangents be equally inclined to the Horizon. In order to find the Nature of the *Catenaria* in any Hypothesis of Gravity, suppose the Gravity to be increased or diminished in the same Proportion as the Thickness of the Chain varies, and to have its Direction changed into the opposite Direction; then imagine a Body to set out with a just Velocity from a given Point in the Chain, and to describe the Curve. The Tension of the Chain at any Point will be always as the Square of the Velocity acquired at that Point, and if a Body be projected with this Velocity in the Direction of the Tangent, the Curvature of the Trajectory described by it will be one Half of the Curvature of the Chain at that Point. We must refer to the Book for a fuller Account of these and of other Theorems.

In the XIIIth Chapter, the Problems concerning the Lines of swiftest Descent, the Figures which amongst all those that have equal Perimeters produce

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Maxima or *Minima*, and the Solid of least Resistance, are resolved without Computations, from the first Fluxions only. There are also easy synthetic Demonstrations subjoined, because this Theory is commonly esteemed of an abstruse Nature, and Mistakes have been more frequently committed in the Prosecution of it, than of any other relating to Fluxions. To give some Idea of the Author's Method, suppose the Gravity to act in parallel Lines, a to denote the Velocity acquired at the lowermost Point of the Curve; and u the Velocity acquired at any other Point of the Curve. Suppose the Element of the Curve to be described by this Velocity u , but the Element of the Base to be always described by the constant Velocity a . Then it is easily demonstrated without any Computation, that the Element of the Ordinate being given, the Difference of the Times in which the Elements of the Curve and Base are thus described is a *Minimum*, when the *Ratio* of those Elements is that of a to u ; *i. e.* when the Sine of the Angle, in which the Ordinate intersects the Curve, is to the *Radius* in this *Ratio*. Supposing therefore this Property to take place over all the Curve, the Excess of the Time in which it is described by the Body descending along it, above the Time in which the Base is described uniformly with the Velocity a , must be a *Minimum*; and this latter Time being given, it follows that the Time of Descent in this Curve is a *Minimum*. When the Gravity tends to a given Centre, substitute an Arc of a Circle described from that Centre through the lowermost Point of the Curve in the Place of the Base in the former Case; and the Property of the Line of swiftest Descent will be discovered in the

same manner. The Nature of the Line that among all those of the same Perimeter is described in the least Time, is discovered with great Facility, by determining from the former Case the Property of the Figure when the Sum or Difference of the Time in which it is described by the descending Body, and of the Time in which it would be described by any given uniform Motion, is a *Minimum*; for the latter Time being the same in all Curves of the same Length, it follows that the Figure, which has this Property, must be described in less Time than any other of an equal Perimeter. The general Isoperimetrical Problems are resolved, and the Solutions are rendered more general, with like Facility by the same Method; which is also applied for determining the Property of the Solid of least Resistance, and serves for resolving the Problem, when Limitations are added concerning the Capacity of the Solid, or the Surface that bounds it.

The last Chapter of the First Book treats chiefly of Gravitation towards Spheroids, of the Figure of the Planets, and of the Tides. The Author, having Occasion in those Inquiries for several new Properties of the Ellipse, begins this Chapter by deriving its Properties from those of the Circle, by considering it as the oblique Section of a Cylinder, or as the Projection of the Circle by parallel Rays upon a Plane oblique to the Circle. In this manner the Properties are briefly transferred from the one to the other, because by this Projection the Centre of the Circle gives the Centre of the Ellipse; Diameters perpendicular to each other in the Circle with their Ordinates, and the circumscribed Square, give conjugate Diameters of the

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the Ellipse with their Ordinates, and the circumscribed Parallelogram; parallel Lines in the Plane of this Circle are projected by Parallels in the Plane of the Ellipse that are in the same *Ratio*; any Area in the former is projected by an Area in the latter, which is in an invariable *Ratio* to it; and concentric Circles give similar concentric Ellipses. It is likewise shewn how Properties of a certain kind are briefly transferred from the Circle to any conic Section with the same Facility.

After demonstrating the Properties of the Ellipse, it is shewn, that if the Gravity of any Particle of a Spheroid being resolved into two Forces, one perpendicular to the Axis of the Solid, the other perpendicular to the Plain of its Equator, then all Particles, equally distant from the Axis, must tend towards it with equal Forces; and all Particles at equal Distances from the Plain of the Equator, gravitate equally towards this Plain; but that the Forces with which Particles at different Distances from the Axis tend towards it, are as the Distances; and that the same is to be said of the Forces with which they tend towards the Plain of the Equator.

From this it is demonstrated, that when the Particles of a fluid Spheroid of an uniform Density gravitate towards each other with Forces that are inversely as the Squares of their Distances, and at the same time any other Powers act on the Particles, either in right Lines perpendicular to the Axis, that vary in the same Proportion as the Distances from the Axis, or in right Lines perpendicular to the Plain of the Equator, that vary as their Distances from it; or when any Powers act on the Particles of the Spheroid, that may be resolved into Forces of this kind; then the
Fluid

Fluid will be every-where *in Equilibrio*, if the whole Force that acts at the Pole be to the whole Force that acts at the Circumference of the Equator, as the Semidiameter of the Equator to the Semiaxis of the Spheroid; and that the Forces with which equal Particles at the Surface tend towards the Spheroid, will be in the same Proportion as Perpendiculars to its Surface, terminated either by the Plane of the Equator, or by the Axis. Because the centrifugal Force with which any Particle of the Spheroid endeavours to recede from its Axis, in consequence of the diurnal Rotation, is as the Distance from the Axis, it appears that if the Earth, or any other Planet, was fluid, and of an uniform Density, the Figure which it would assume would be accurately that of an oblate Spheroid generated by an Ellipsis revolving about its Second Axis.

Afterwards the Gravity towards an oblate Spheroid is accurately measured by circular Arcs, not only at the Pole, but also at the Equator, and in any intermediate Places; and the Gravity towards an oblong Spheroid is measured by Logarithms. The Gravity at any Distance in the Axis of the Spheroid, or in the Plane of the Equator produced, is likewise accurately determined by like Measures, without any new Computation or Quadrature, by shewing that when Two Spheroids have the same Centre and *Focus*, and are of an uniform Density, the Gravities towards them at the same Point in the Axis or Plane of the Equator produced, are as the Quantities of Matter in the Solids.

This Theory is applied for determining the Figure of the Earth, by comparing the Force of Gravity in
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any given Latitude, derived from the Length of a *Pendulum* that vibrates there in a Second of Time, with the centrifugal Force at the Equator, deduced from the periodic Time of the diurnal Rotation, and the Amplitude of a Degree of the Meridian; or by comparing the Lengths of *Pendulums* that vibrate in equal Times in given unequal Latitudes; or by comparing different Degrees measured upon the Meridian. By the best Observations it would seem, that there is a greater Increase of Gravitation, and of the Degrees of the Meridian from the Equator towards the Poles, than ought to arise from the Supposition of an uniform Density. Therefore the Author supposes the Density to vary from the Surface towards the Centre; and, in several Cases he has considered, he finds that a greater Density towards the Centre would account for a greater Increase of Gravitation towards the Poles, but not for a greater Increase of the Degrees of the Meridian; and that the Hypothesis of a less Density towards the Centre would account for the latter, but not for the former, supposing (after Sir *Isaac Newton*) the Columns of the Fluid to extend from the Surface to the Centre, and there to sustain each other. On this Account he determines the Gravitation towards the Earth, when it is supposed to be hollow with a *Nucleus* included, according to the Hypothesis advanced by Dr. *Halley*, with the Difference of the Semidiameters that might arise from such a Disposition of the internal Parts. But in this Case, and when the Density is supposed variable, the spheroidal Figure is only assumed as an Hypothesis. He adds, that by imagining the Density to be greater in the Axis than in the Plain of the Equator at equal Di-

Distances from the Centre, an Hypothesis perhaps might be found, that would account for most of the *Phænomena*; but that a Series of many exact Observations is requisite, before we can examine with any Certainty the various Suppositions that may be imagined concerning the internal Constitution of the Earth. This Doctrine is likewise applied for determining the Figure of *Jupiter*.

It follows from the same Theorem, that if we suppose the Earth to be fluid, and abstract from its Motion upon its Axis, and the Inclination of the right Lines in which its Particles gravitate towards the Sun or Moon, the Figure which it would assume in consequence of the unequal Gravitation of its Particles towards either of those Bodies would be accurately that of an oblong Spheroid having its Axis directed towards that Body. The Ascent of the Water, deduced from this Theorem, agrees nearly with that which Sir *Isaac Newton* found, by computing it briefly from what he had demonstrated concerning the Figure of the Earth. Several Observations are subjoined concerning the Tides, and the Causes which may contribute to increase or diminish them, particularly the Inequality of the Velocities with which Bodies revolve about the Axis of the Earth in different Latitudes.

This Chapter concludes by demonstrating briefly, that if the Attraction of the Particles decreased as the Cube of their Distance increases, or in any higher Proportion, then any Particle would tend towards the least Portion of Matter in Contact with it, with a greater Force than towards the greatest Body at any Distance, how small soever from it. The true
Law

Law of Gravity is better adapted for holding the Parts of each Body in a proper Union, while it perpetuates the Motions in the great System about the Sun, and preserves the Revolutions in the lesser Systems nearly regular; and the Author concludes with observing, that a remarkable geometrical Simplicity is often found in the Conclusions that are derived from it.

An Account of Book II. will be given in the next Transaction.

VII. *De Calculo prægrandi a Muliere cum Urina excreto Observatio Dⁿⁱ Antonii Leprotti, R. S. S. Pont. Max. Archiat. per Abbatem Didacum de Revillas, R. S. S. ad D. Smart Lethieullier R. S. S. transmissa.*

Read Jan. 27.
1742-3.

Romæ, pridie Cal. Januarii An. 1743.
QUÆ quindecim abhinc annis Vidua est Mulier pauper quinquagesimum agens annum summa urinæ difficultate per quadraginta Menses laborabat; quum nocte ei supervenit mictus, imo sinceri cruoris profluvium ad tres circiter libras; simulque Lapis, ejus, quæ in adjecta Figura describitur, formæ & molis, extrusus est, cujus pondus jam exsiccati uncias duas & grana novem ac viginti exæquat*. Mulier autem ingenti per eos menses gravitatis sensu, assiduoque dolore, ad Vesicæ cervicem afficiebatur sive cubans sive crecta; nunc autem solido exacto mense, ea invita, prodere pergunt cum sanie urinæ. *Vide Fig. II. in TAB. I.*

unc. den. gr.

* i. e. 1 : 17 : 4. TR.